Received: *June 30, 2000*, Accepted: *August 17, 2000*
Revised: *August 9, 2000*

Multi-field fermionic preheating

Shinji Tsujikawa
Department of Physics, Waseda University, 3-4-1 Ohkubo, Shinjuku-ku, Tokyo 169-8555, Japan
E-mail: shinji@gravity.phys.waseda.ac.jp

Bruce A. Bassett
Relativity and Cosmology Group (RCG), University of Portsmouth, Mercantile House, Portsmouth, PO1 2EG, England
E-mail: bruce.bassett@port.ac.uk

Fermin Viniegra
Department of Theoretical Physics, Oxford University Oxford OX1 3NP, England
E-mail: fermin@thphys.ox.ac.uk

Abstract: Fermion creation during preheating in the presence of multiple scalar fields exhibits a range of interesting behaviour. We present non-perturbative analysis of fermion production over a 6-dimensional parameter space in an expanding background paying particular attention to the interplay between instant preheating (χ-ψ) and direct fermion preheating (ϕ-ψ). In the broad resonance regime we find that instant fermion production is sensitive to suppression of long wavelength χ modes during inflation. Further, the standard scenario of resonant fermionic preheating through inflaton decay can be significantly modified by the χ-ψ coupling, and may even lead to a decrease in the number of fermions produced. We explicitly include the effects of metric perturbations and demonstrate that they are important at small coupling but not at strong coupling, due to the rapid saturation of the Pauli bound.

Keywords: Cosmology of Theories beyond the SM, Physics of the Early Universe, Cosmological Phase Transitions, Classical Theories of Gravity.

Contents

1. Introduction

The evolution of spin 1/2 (and higher) quantum fields propagating on curved backgrounds has traditionally been considered an esoteric domain of little cosmological interest [1]. In the face of the power and simplicity of the inflationary paradigm the added complexities associated with their renormalization, regularization and solution [2, 3] in the presence of a non-trivial metric $g_{\mu\nu}$ made them unpopular topics of study.

The last two years have, however, seen a reversal in the fortune of higher-spin fields with the growing belief that they may be relevant for cosmology in a number of rather profound ways. Decay of the inflaton to massive states *during inflation* [4] can lead to non-trivial features in the primordial power spectrum and cosmic microwave background (CMB) [5]. In inflationary scenarios it is during reheating and preheating — the non equilibrium end to many inflationary models — that fermions first became important for the subsequent evolution and nature of the universe.

There are three types of inflaton decay products of significant current interest:

(i) gravitinos, the supersymmetric partner of the graviton present in supergravity (SUGRA) [6],

(ii) heavy right-handed neutrinos which may have mediated lepto-genesis [7] and

(iii) the so-called *cryptons* [8] or *wimpzillas* [9] — massive ($\sim 10^{12}\,\mathrm{GeV}$) dark matter which may account for the cosmic rays observed beyond the Greisen-Zatsepin-Kuzmin (GZK) cut-off.

While both heavy right-handed neutrinos and cryptons provide potential solutions to serious problems, the spin-3/2 gravitini are a plague. A typical constraint is that their number density $n_{3/2}$ must satisfy the bound $n_{3/2}/s \lesssim 10^{-14}$ where s is the entropy density of the universe at reheating for a gravitino of mass $m_{3/2} \sim \mathcal{O}(100)\,\mathrm{GeV}$.

In a perturbative theory of reheating [10], this leads to the constraint $T_r \lesssim 10^8\,\mathrm{GeV}$ [11, 6], on the reheat temperature, T_r. Early studies of gravitino production [12] following preheating [13]–[16] argued that thermal gravitino abundances were significantly enhanced due to the non-perturbative nature of preheating. Nevertheless, it was only recently realized that cosmological bounds arising from preheating are much tighter than previously imagined [17]–[23].

In these latter works the direct production of gravitini was studied through their coupling to the inflaton, ϕ. Nevertheless, in a general SUGRA theory one expects many scalar fields ϕ_i to contribute to the Kähler potential $K(\phi_i\phi^{i*})$. Since the gravitino effective mass is given by [18]

$$m_{3/2}(\phi_i) = e^{K/2}\frac{W}{m_{\rm pl}^2}\,, \tag{1.1}$$

where W is the superpotential and $m_{\rm pl} \sim 10^{19}\,\mathrm{GeV}$ is the Planck mass, it is natural to ask how other scalar fields will affect gravitino production.

Since we are interested in general results valid also for leptogenesis scenarios we will not work directly with the gravitino (Rarita-Schwinger) equations but rather with the spin 1/2 Dirac equation [24]. The spin 3/2 gravitino has both helicity-3/2 and 1/2 states. On the one hand the helicity-3/2 mode reduces directly to a Dirac equation [3, 17] and on the other, the dangerous helicity-1/2 mode, which is not suppressed in the $m_{\rm pl} \to \infty$ limit [18], behaves like the goldstino in the high momentum limit [21, 22, 23].

We will consider a rather general potential leading to a fermion effective mass

$$m_{\rm eff} = m_\psi + h_1\phi + h_2\chi\,, \tag{1.2}$$

where m_ψ is the bare mass of fermion, ϕ and χ are the inflaton and a scalar field coupled to inflaton, respectively. The addition of the h_2 coupling modifies the standard production in two interesting cases (i) if $\langle\chi\rangle \neq 0$ during inflation or (ii) if χ itself undergoes resonant amplification during preheating. We will consider both cases.

While we will draw general conclusions regarding multi-field resonant fermion production there are certain issues we will not address. Equation (1.2) does not model the production of the helicity-1/2 mode of the gravitino since, even in the single chiral field case, this involves derivatives of the inflaton [19, 23]. In the general multi-chiral case the gravitino will also be coupled to the other fermionic degrees of freedom implicit in the system.

Further, from eq. (1.2) it is clear that in general the fermion effective mass will have more than one natural frequency, analogous to the situation with scalar fields. If the natural frequencies (of ϕ and χ) are irrationally related then the effective mass is quasi-periodic [25]. In the scalar case this causes a dramatic increase in the strength and breadth of the resonance bands.

Similarly, if the fermion is coupled to many scalar fields the effective mass (1.2) may become chaotic or stochastic. In the scalar case this is known to again enhance the resonance if the time correlations are small [26]. Whether these effects occur in the fermion case too is unknown at present.

Instead in this work we will investigate in detail the following issues:

- What is the interplay between direct (through the coupling h_1) and instant (through h_2) [27] fermion preheating ?
- How is instant preheating sensitive to the χ vacuum expectation value (VEV) during inflation? This will strongly alter the fermion effective mass, eq. (1.2), and therefore the nature of stochastic fermion production.
- How does the inclusion of metric perturbations affect fermion production? Are their effects similar to the scalar case?

To answer these issues requires a 6-dimensional parameter space (illustrated in figure 1). We then study the evolution of the spectrum of massive spin-1/2 fermions over this "moduli" space of fermionic preheating.

In section 2 we discuss this parameter space and present the equations for the background geometry and perturbations. Fermionic preheating is then analysed in the massless (section 3) and massive cases (section 4).

2. The model and basic equations

Let us consider the massless chaotic inflation model

$$V = \frac{1}{4}\lambda\phi^4 + \frac{1}{2}g^2\phi^2\chi^2 + \tilde{g}^2\phi^3\chi + (m_\psi + h_1\phi + h_2\chi)\overline{\psi}\psi\,, \tag{2.1}$$

where ψ is the fermion field of interest with bare mass m_ψ, which is coupled to the inflaton, ϕ, and a scalar field χ through the couplings h_1 and h_2, respectively. Typically we adopt the value $\lambda = 10^{-12}$ for the self-coupling which comes from CMB constraints.

The perturbative (tree-level) decay rates for the inflaton and the χ field to fermions are [10]

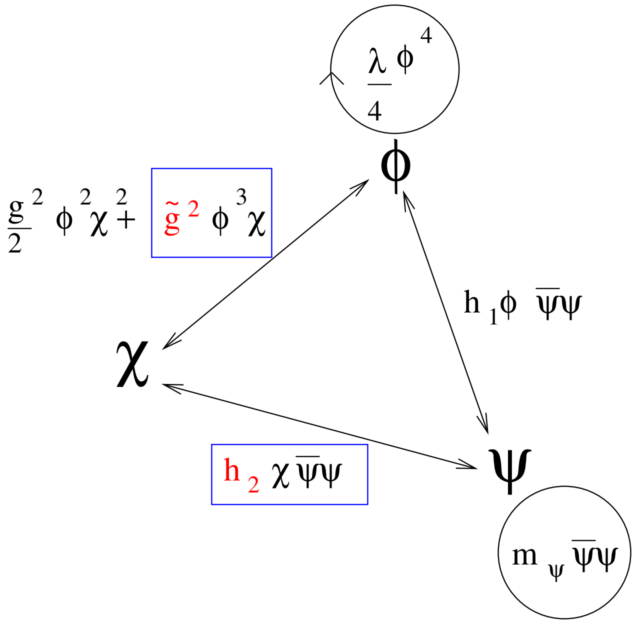

Figure 1: Schematic illustration of the potential (2.1). We consider the inflaton ϕ, scalar field χ and fermion ψ. Interactions are shown as lines with arrows, leading to a 6-dimensional parameter space of couplings and masses. The new couplings we consider in this paper are shown in red and are boxed.

$$\begin{aligned}
\Gamma(\phi \to \psi\psi) &= \frac{h_1^2 m_\phi}{8\pi} = \frac{h_1^2}{8\pi}\left(\lambda\phi^2 + g^2\chi^2 + 3\tilde{g}^2\phi\chi\right)^{1/2}, \\
\Gamma(\chi \to \psi\psi) &= \frac{h_2^2 m_\chi}{8\pi} = \frac{h_2^2 g|\phi|}{8\pi}. \qquad (2.2)
\end{aligned}$$

However, these decay rates cease to be accurate during resonance and after a fermion-scalar plasma has formed. Boyanovsky et al. [28, 29], Baacke et al. [30, 31] and Ramsey et al. [32] studied this resonant, non-equilibrium production of fermions in the absence of the χ field.

Greene & Kofman [33] gave analytical insight into resonant fermion production and recent work has brought out the strong analogy with the scalar stochastic resonance regime [34]–[37]. We extend these works to the multi scalar field case where the χ field is coupled to both the ψ and ϕ fields.

We have also included the $\tilde{g}^2\phi^3\chi$ coupling which may for example, appear in SUGRA models [38] in addition to the usual $\frac{1}{2}g^2\phi^2\chi^2$ coupling. Due to the existence of this term, $\langle\chi\rangle \neq 0$ during inflation and the exponential suppression for the χ field during inflation is avoided [39, 40]. We will see later that $\tilde{g}$ can significantly enhance the χ-ψ decays.

When $m_\psi = 0$, the complete system of equations is conformally invariant (modulo the small conformal couplings needed for ϕ and χ). Conformally rescaled fields will be capped with a $\tilde{}$, i.e. $\tilde{\phi} \equiv a\phi$, $\tilde{\chi} \equiv a\chi$, and $\tilde{\psi} \equiv a^{3/2}\psi$ with a the scale factor.

2.1 The background and scalar field equations

In this paper, we work around a flat Friedmann-Lemaître-Robertson-Walker (FLRW) background with perturbations in the longitudinal gauge

$$ds^2 = -(1+2\Phi)dt^2 + a^2(1-2\Psi)\delta_{ij}dx^i dx^j\,, \tag{2.3}$$

where Φ and Ψ are gauge-invariant potentials [41]–[44]. The relation $\Phi = \Psi$ follows since the anisotropic stress vanishes at linear order [44]. Decomposing the scalar fields into homogeneous parts and fluctuations $\phi(t,\mathbf{x}) \to \phi(t) + \delta\phi(t,\mathbf{x})$, $\chi(t,\mathbf{x}) \to \chi(t) + \delta\chi(t,\mathbf{x})$, one obtains the equations for the Fourier modes of the scalar field perturbations:

$$\begin{aligned}
\delta\ddot{\phi}_k + 3H\delta\dot{\phi}_k + \left(\frac{k^2}{a^2} + 3\lambda\phi^2 + g^2\chi^2 + 6\tilde{g}^2\phi\chi\right)\delta\phi_k = \\
= 4\dot{\phi}\dot{\Phi}_k + 2(\ddot{\phi} + 3H\dot{\phi})\Phi_k - (2g^2\phi\chi + 3\tilde{g}^2\phi^2)\delta\chi_k\,, \qquad (2.4)\\
\delta\ddot{\chi}_k + 3H\delta\dot{\chi}_k + \left(\frac{k^2}{a^2} + g^2\phi^2\right)\delta\chi_k = \\
= 4\dot{\chi}\dot{\Phi}_k + 2(\ddot{\chi} + 3H\dot{\chi})\Phi_k - (2g^2\phi\chi + 3\tilde{g}^2\phi^2)\delta\phi_k\,. \qquad (2.5)
\end{aligned}$$

A convenient equation for the potential Φ_k is

$$\dot{\Phi}_k + H\Phi_k = 4\pi G(\dot{\phi}\delta\phi_k + \dot{\chi}\delta\chi_k)\,, \tag{2.6}$$

where $G \equiv m_{\rm pl}^{-2}$ is Newton's gravitational constant. In eq. (2.6), we neglect the contribution of fermions since it is second order and typically negligible relative to the scalar fields due to the Pauli exclusion principle.

We include the second order energy densities in the scalar field fluctuations in the evolution of the background quantities via k-space integrals. The equations for the Hubble parameter ($H \equiv \dot{a}/a$) and homogeneous components of the scalar fields are thus given by [45]

$$\begin{aligned}
H^2 = \frac{8\pi G}{3}\Bigg[\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\langle\delta\dot{\phi}^2\rangle + \frac{1}{2a^2}\langle(\nabla\delta\phi)^2\rangle + \frac{1}{4}\lambda\left(\phi^4 + 6\phi^2\langle\delta\phi^2\rangle + 3\langle\delta\phi^2\rangle^2\right) + \\
+ \frac{1}{2}\dot{\chi}^2 + \frac{1}{2}\langle\delta\dot{\chi}^2\rangle + \frac{1}{2a^2}\langle(\nabla\delta\chi)^2\rangle + \frac{g^2}{2}\langle\phi^2\chi^2\rangle + \tilde{g}^2\langle\phi^3\chi\rangle\Bigg]\,, \qquad (2.7)\\
\ddot{\phi} + 3H\dot{\phi} + \lambda\phi(\phi^2 + 3\langle\delta\phi^2\rangle) + g^2(\chi^2 + \langle\delta\chi^2\rangle)\phi + 3\tilde{g}^2(\phi^2 + \langle\delta\phi^2\rangle)\chi = 0\,, \qquad (2.8)\\
\ddot{\chi} + 3H\dot{\chi} + g^2(\phi^2 + \langle\delta\phi^2\rangle)\chi + \tilde{g}^2(\phi^3 + 3\phi\langle\delta\phi^2\rangle) = 0\,, \qquad (2.9)
\end{aligned}$$

where the expectation values of $\delta\phi^2$ and $\delta\chi^2$ are defined as

$$\begin{aligned}
\langle\delta\phi^2\rangle &= \frac{1}{2\pi^2}\int k^2|\delta\phi_k|^2 dk\,, \qquad (2.10)\\
\langle\delta\chi^2\rangle &= \frac{1}{2\pi^2}\int k^2|\delta\chi_k|^2 dk\,. \qquad (2.11)
\end{aligned}$$

As for other definitions of the fluctuational quantities, see e.g. ref. [46]. In the present model, both $\langle \delta\phi^2 \rangle$ and $\langle \delta\chi^2 \rangle$ grow by parametric resonance as shown in figure 5, and alter the evolution of the background fields.

We implement backreaction using the Hartree approximation (i.e. neglecting rescattering effects [15]) and ignore the backreaction effect due to the production of fermions which is typically justified because of the small amount of energy in the fermions by the exclusion principle (very massive fermion production being an obvious example where this is not necessarily true). The Hartree approximation alters the terms ϕ^2 and χ^2 to $\langle \phi^2 \rangle \equiv \phi^2 + \langle \delta\phi^2 \rangle$ and $\langle \chi^2 \rangle \equiv \chi^2 + \langle \delta\chi^2 \rangle$ in eqs. (2.4) and (2.5), respectively. The variances are crucial to ensure conservation of energy.

2.2 The Dirac equation

The Dirac equation in curved spacetime is given in general by [1]

$$i\gamma^\mu \left(\partial_\mu + \frac{1}{8} \left[\gamma^b, \gamma^c \right] e_b^\nu e_{c\nu;\mu} \right) \psi - m_{\text{eff}} \psi = 0 \,, \tag{2.12}$$

where the Latin indices a, b, c, and Greek indices μ, ν refer to the locally inertial (tetrad) coordinates and the general curvilinear (spacetime) coordinates, respectively. The vierbeins e_a^μ connect these coordinate systems via

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \eta_{ab} e^a e^b \,, \tag{2.13}$$

where $g_{\mu\nu}$ and η_{ab} are the curved space and Minkowskian metrics, respectively. The γ^μ are the curved-space Dirac matrices satisfying the Clifford algebra anti-commutator relation $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$.

Since the spinor ψ has no homogeneous component, writing the general Dirac equation to first order in the perturbed metric (2.3) does not yield any terms containing Φ_k [in contrast with the scalar case, eqs. (2.4) and (2.5)] and simply gives

$$(i\gamma^\mu \partial_\mu - m_{\text{eff}} a) \tilde{\psi} = 0 \,, \tag{2.14}$$

where ∂_0 denotes the derivative with respect to conformal time $\eta = \int a^{-1} dt$. We decompose the $\tilde{\psi}$ field into Fourier components as

$$\tilde{\psi} = \frac{1}{(2\pi)^{3/2}} \int d^3k \sum_s \left[a_s(k) \tilde{\mathbf{u}}_s(k,\eta) e^{+i\mathbf{k}\cdot\mathbf{x}} + b_s^\dagger(k) \tilde{\mathbf{v}}_s(k,\eta) e^{-i\mathbf{k}\cdot\mathbf{x}} \right] . \tag{2.15}$$

Imposing the following standard ansatz [33]:

$$\tilde{\mathbf{u}}_s(k,\eta) = (-i\gamma^\mu \partial_\mu - m_{\text{eff}} a) \tilde{\psi}_k(t) R_\pm(k) \,, \tag{2.16}$$

where $R_\pm(k)$ are the eigenvectors of the helicity operator, which satisfy the relation $\gamma^0 R_\pm(k) = 1$ and $\mathbf{k} \cdot \sum R_\pm(k) = \pm 1$, we obtain the mode equation for the $\tilde{\psi}_k$:

$$\left[\frac{d^2}{d\eta^2} + k^2 + (m_{\text{eff}} a)^2 - i \frac{d}{d\eta} (m_{\text{eff}} a) \right] \tilde{\psi}_k = 0 \,. \tag{2.17}$$

JHEP08(2000)019

We introduce dimensionless quantities

$$f \equiv \frac{\tilde{m}_{\text{eff}}}{\sqrt{\lambda}\phi(0)}\,, \qquad \kappa^2 \equiv \frac{k^2}{\lambda\phi^2(0)}\,, \qquad x \equiv \sqrt{\lambda}\phi(0)\eta\,, \tag{2.18}$$

with

$$\tilde{m}_{\text{eff}} \equiv a m_{\text{eff}} = a m_\psi + h_1\tilde{\phi} + h_2\tilde{\chi}\,. \tag{2.19}$$

Then eq. (2.17) becomes

$$\left[\frac{d^2}{dx^2} + \kappa^2 + f^2 - i\frac{df}{dx}\right]\tilde{\psi}_k = 0\,, \tag{2.20}$$

which has a WKB-form solution given by

$$\tilde{\psi}_k = \alpha_k N_+ e^{-i\int_0^t \Omega_k dx} + \beta_k N_- e^{+i\int_0^t \Omega_k dx}\,, \tag{2.21}$$

where $\Omega_k^2 \equiv \kappa^2 + f^2$ and $N_\pm \equiv 1/\sqrt{2\Omega_k(\Omega_k \pm f)}$. The comoving number density of produced fermions is given in terms of the Bogoliubov coefficients by

$$n_k \equiv |\beta_k|^2 = \frac{1}{2} - \frac{\kappa^2}{\Omega_k}\,\mathrm{Im}\left(\tilde{\psi}_k \frac{d\tilde{\psi}_k^*}{dx}\right) - \frac{f}{2\Omega_k}\,. \tag{2.22}$$

The initial conditions are chosen to be $\alpha_k(0) = 1$, $\beta_k(0) = 0$, which corresponds to $n_k(0) = 0$. The Bogoliubov coefficients satisfy the relation $|\alpha_k(t)|^2 + |\beta_k(t)|^2 = 1$, which means that the exclusion principle restricts the number density of fermions to below unity, $n_k(t) \le 1$. In the subsequent sections, we investigate the dynamics of fermionic preheating taking into account both the h_1 and h_2 coupling.

3. Massless fermion production

It was pointed out by several authors [30, 33] that *massless* fermions are resonantly produced by the interaction between the coherently oscillating inflaton and fermion field during preheating. The case of most interest $q_1 \equiv h_1^2/\lambda \gg 1$ leads to strongly non-adiabatic evolution ($\dot{\Omega}_k/\Omega_k^2 \gg 1$) of the $\tilde{\psi}_k$ only for the short periods when $\tilde{m}_{\text{eff}} \simeq 0$. For $h_2 = 0$, this coincides with $\phi = 0$ and leads to efficient fermion production for momenta $\kappa \lesssim q_1^{1/3}$.

In most previous works of fermion production, the interaction between ψ and χ fields has been neglected. However, since the χ field can be amplified during preheating, it is expected that this will lead to the fermion production even in the absence of the direct (h_1) coupling.

The strength of the χ field resonance depends on g. As was analytically studied in [45], the strongest resonance occurs around $\kappa = 0$ when $g^2/\lambda = 2l^2$ where l is an integer. Hence χ particle production is maximally efficient even if $g^2/\lambda = \mathcal{O}(1)$, which is different from the massive inflation model where efficient preheating typically requires large couplings $g \gtrsim 3 \times 10^{-4}$.

JHEP08(2000)019

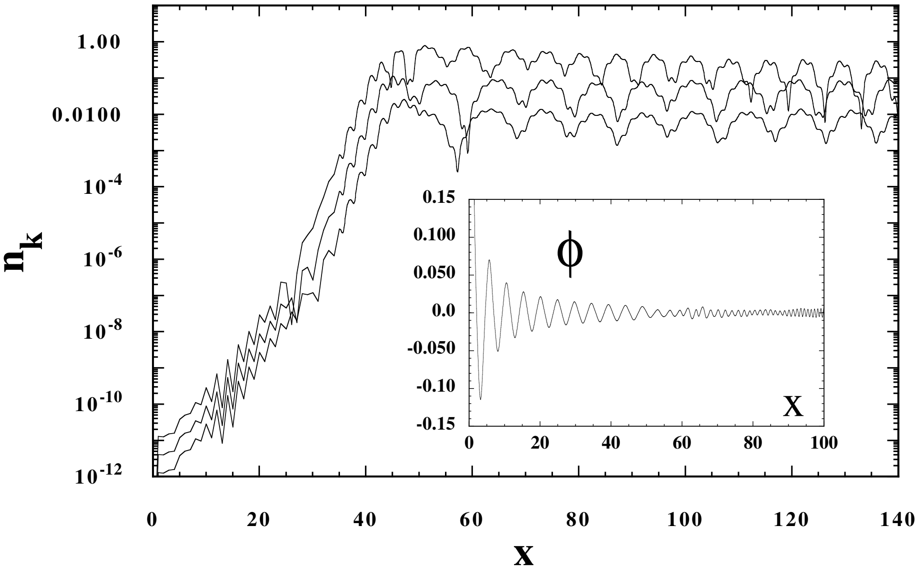

Figure 2: The evolution of the comoving number density of fermions n_k versus dimensionless conformal time x during preheating in the case of $q_2 = 1, 10, 100$ (bottom, middle, top) and $q_1 = 0$ with $g^2/\lambda = 2$ and $\tilde{g} = 0$ for the mode $\kappa = 1$. The number density grows exponentially until the growth of the χ fluctuation stops by backreaction effects. We choose the initial values of scalar fields to be $\phi(0) = 0.5\, m_{\rm pl}$ and $\chi(0) = 10^{-9}\, m_{\rm pl}$, which we use in all figures in which $\tilde{g} = 0$. **Inset**: the evolution of the inflaton condensate, ϕ, vs x. The backreation effect becomes relevant for $x \gtrsim 50$.

3.1 light χ field: $g^2/\lambda = \mathcal{O}(1)$

Let us first consider fermions with vanishing bare mass ($m_\psi = 0$) with $g^2/\lambda = 2$ and $\tilde{g} = 0$, which leads to largest growth rate (Floquet index) for the super-Hubble $\kappa \to 0$ modes. We have numerically solved the background and perturbed equations and in figure 2 the evolution of the comoving number density of fermions. Plotted are the cases of $q_2 \equiv h_2^2/\lambda = 1, 10, 100$ and $q_1 = 0$ for the mode $\kappa = 1$. We start integrating from the end of inflation, and take the initial scalar field values to be $\phi(0) = 0.5 m_{\rm pl}$ and $\chi(0) = 10^{-9} m_{\rm pl}$. Since the χ field grows exponentially through parametric resonance, this changes the frequency of the fermion field even in the absence of the h_1 coupling.

In fact, n_k increases stochastically until $x \approx 50$ due to the rapid growth of the χ field. Not surprisingly we find in figure 2 that increased coupling h_2 leads to larger occupation numbers. For couplings $q_2 \gtrsim 100$, the final number density reaches of order unity and further increases ($q_2 \gg 100$) cause n_k to increase more rapidly.

For small momentum modes $\kappa \sim 0$, the excitation of fermions is stronger than the sub-Hubble mode in figure 2 because the non-adiabatic change of the fermion frequency is more relevant. After χ particle production stops by backreaction effects, the fermion production also terminates, but does not oscillate.

JHEP08(2000)019

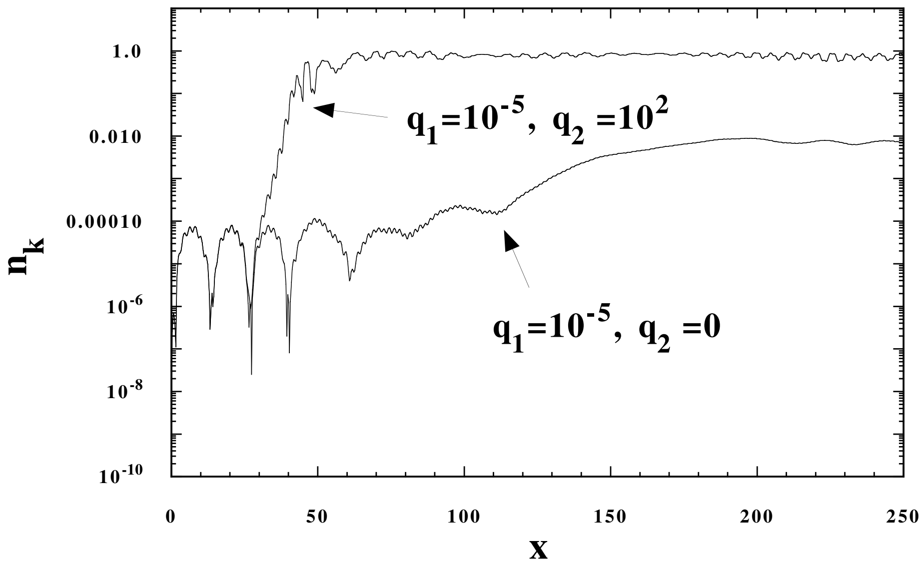

Figure 3: The evolution of the comoving number density of fermions n_k during preheating in the cases of $q_1 = 10^{-5}, q_2 = 10^2$ (top) and $q_1 = 10^{-5}$, $q_2 = 0$ (bottom) with $g^2/\lambda = 2$ and $\tilde{g} = 0$ for the mode $\kappa = 1$. The initial values of the scalar fields are $\phi(0) = 0.5m_{\rm pl}$ and $\chi(0) = 10^{-9}m_{\rm pl}$. The existence of the h_2 coupling clearly assists fermion production in the later phases of preheating once the χ field has grown significantly through resonance.

We should mention that the final occupation number of fermions depends not only on h_2 but also on the value of χ at the onset of preheating. In the case of $g^2/\lambda = \mathcal{O}(1)$, since the $g\phi$ term in the l.h.s. of eq. (2.9) is comparable to the Hubble rate, the χ field is hardly damped during inflation [47]. Moreover, even for an initial value of $\chi(0) = 10^{-9}m_{\rm pl}$, which is much smaller than $\phi(0)$, fermions can be created purely by the h_2 coupling.

When the interaction, mediated by h_1, between inflaton and fermion is taken into account, fermion production at the initial stage is mainly dominated by the h_1 coupling unless $h_2 \gg h_1$ because χ is so much smaller than ϕ at the beginning of preheating. However, parametric amplification of the χ field leads to an additional growth of the occupation number of fermions.

In figure 3, we plot the evolution of n_k for the cases of $q_1 = 10^{-5}, q_2 = 10^2$ and $q_1 = 10^{-5}, q_2 = 0$ with $g^2/\lambda = 2$ for the mode $\kappa = 1$. We find that the h_2 coupling assists fermion production for $x \gtrsim 30$, leading to the final density $n_k \sim 1$. In figure 4, the spectrum of the final number density n_k is depicted. When $q_1 = 10^{-5}$ and $q_2 = 0$, n_k does not reach of order unity except for momenta close to $\kappa = 0$.

In the case of $g^2/\lambda = \mathcal{O}(1)$, χ and super-Hubble $\delta\chi_k$ modes are not damped during inflation [47, 48] and have a nearly flat spectrum, as opposed to a k^3 spectrum when $g^2/\lambda \gg 1$ [54, 55].

This means that when the system enters preheating, the growth of χ and $\delta\chi_k$ stimulate the growth of super-Hubble Φ_k modes as can be seen in eq. (2.6) and in

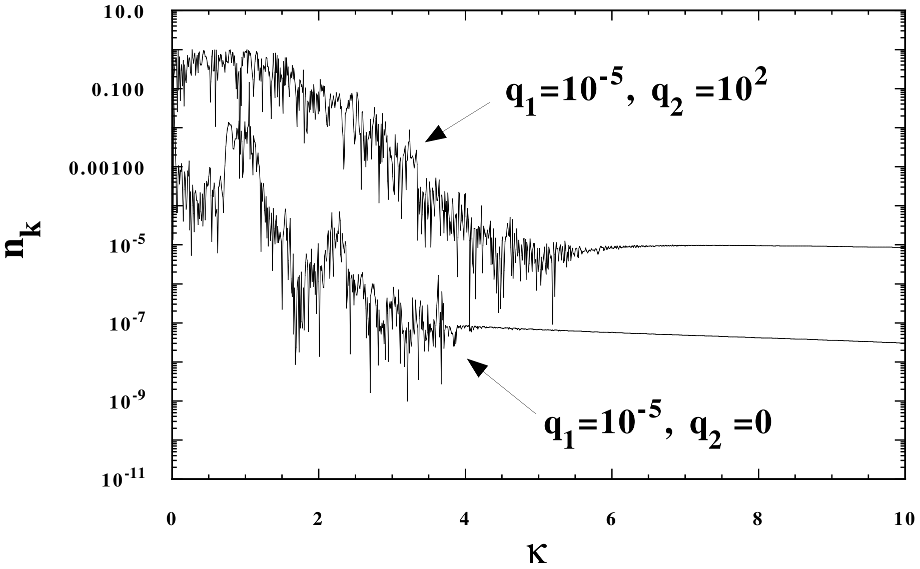

Figure 4: The spectra of the final comoving number density of fermions n_k in the cases of $q_1 = 10^{-5}, q_2 = 10^2$ (top) and $q_1 = 10^{-5}$, $q_2 = 0$ (bottom) with $g^2/\lambda = 2$ and $\tilde{g} = 0$. The initial values of scalar fields are chosen as $\phi(0) = 0.5 m_{\rm pl}$ and $\chi(0) = 10^{-9} m_{\rm pl}$.

figure 5. The growth rate of the $\delta\chi_k$ fluctuation for the low momentum mode $\kappa \sim 0$ is large as $\mu = 0.2377$ for $g^2/\lambda = 2$ [45].

In the standard adiabatic inflation scenario, the Bardeen parameter

$$\zeta_k \equiv \frac{H\left(\dot{\phi} Q_k^{\phi} + \dot{\chi} Q_k^{\chi}\right)}{\dot{\phi}^2 + \dot{\chi}^2}, \tag{3.1}$$

is time-independent in the long-wavelength limit, a result which simply follows from energy conservation [49]. Here $Q_k^{\phi} \equiv \delta\phi_k + \dot{\phi}\Phi_k/H$ and $Q_k^{\chi} \equiv \delta\chi_k + \dot{\chi}\Phi_k/H$ are the Sasaki-Mukhanov variables. However, in the multi-field case where super-Hubble metric perturbations are enhanced, amplification of isocurvature scalar modes leads to amplification of ζ_k on large scales *before backreaction terminates the resonance* as shown in the inset of figure 5.

When $1 < g^2/\lambda < 3$ where super-Hubble modes of the $\delta\chi_k$ fluctuation are enhanced, super-Hubble metric perturbations are amplified unless χ is initially very small. In the simulation of figure 5 where the initial value of χ is chosen as $\chi(0) = 10^{-9} m_{\rm pl}$, super-Hubble Φ_k modes increase to the order of 0.1. When $\chi(0) \gtrsim 10^{-8} m_{\rm pl}$, we numerically find that metric perturbations on large scales grow to the non-linear level $\Phi_k \sim 1$.

In the rigid spacetime case (found by setting $\Phi_k \equiv 0$), the maximum characteristic Floquet exponent for the $\delta\phi_k$ fluctuation is $\mu_{\rm max} = 0.03598$ in the sub-Hubble range $3/2 < k^2/(\lambda\phi_{\rm co}^2) < \sqrt{3}$, where $\phi_{\rm co}$ is the value when the ϕ field begins to oscillate coherently. This growth rate is one order smaller than that for $\delta\chi_k$.

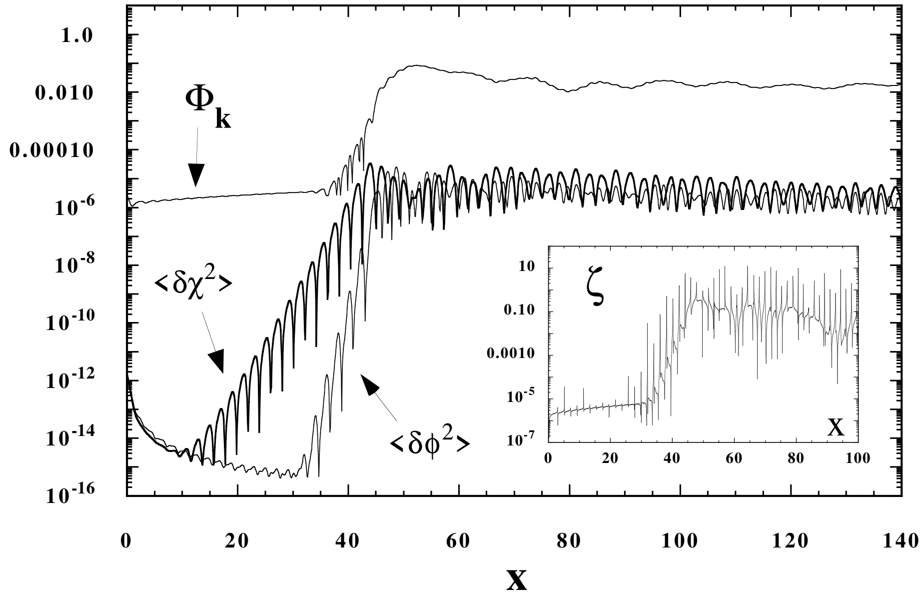

Figure 5: The evolution of the super-Hubble metric perturbation Φ_k with mode $\kappa = 10^{-26}$, and field variances $\langle\delta\chi^2\rangle$ and $\langle\delta\phi^2\rangle$ for $g^2/\lambda = 2$ and $\tilde{g} = 0$. We choose the initial values of scalar fields as $\phi(0) = 0.5m_{\rm pl}$ and $\chi(0) = 10^{-9}m_{\rm pl}$. When the $\delta\chi_k$ fluctuation is sufficiently amplified, both Φ_k and $\delta\chi_k$ begin to grow. Finally, backreaction effects of produced particles shut off the resonance. **Inset**: the evolution of the Bardeen parameter ζ_k for the super-Hubble mode $\kappa = 10^{-26}$. The growth of ζ_k is due to the fact that the χ field is light during inflation for $g^2/\lambda = 2$ [47].

In the realistic perturbed metric ($\Phi_k \neq 0$), however, the r.h.s. of eq. (2.4) leads to the enhancement of the $\delta\phi_k$ fluctuation for super-Hubble modes. This makes the growth rate of $\langle\delta\phi^2\rangle$ comparable to that of $\langle\delta\chi^2\rangle$ as is found in figure 5. The final variance $\langle\delta\phi^2\rangle_f \sim 10^{-5}m_{\rm pl}^2$ is about two orders of magnitude larger than in the rigid spacetime case in the Hartree approximation [50, 51].

The rapid increase of the $\delta\phi_k$ fluctuation causes backreaction to become dynamically important earlier. This means that the final variance of $\langle\delta\chi^2\rangle$, which we denote $\langle\delta\chi^2\rangle_f$, is expected to be larger in the rigid case compared with the perturbed metric case. This is verified numerically where we find that $\langle\delta\chi^2\rangle_f$ is larger in the rigid case by about one order of magnitude.

Clearly this will have an effect on "instant" fermion production due to the h_2 coupling which is modified when metric perturbations are taken into account. While the spectra of fermions before backreaction sets in are almost the same in both the perturbed and unperturbed metric, including metric perturbations reduces the maximum occupation number of fermions as is evident from figures 6 and 7, although this effect is rather weak.

This difference practically disappears in the strong coupling regime $q_2 \gg 1$ due to the saturation of the Pauli bound. While we find some difference in the weak coupling

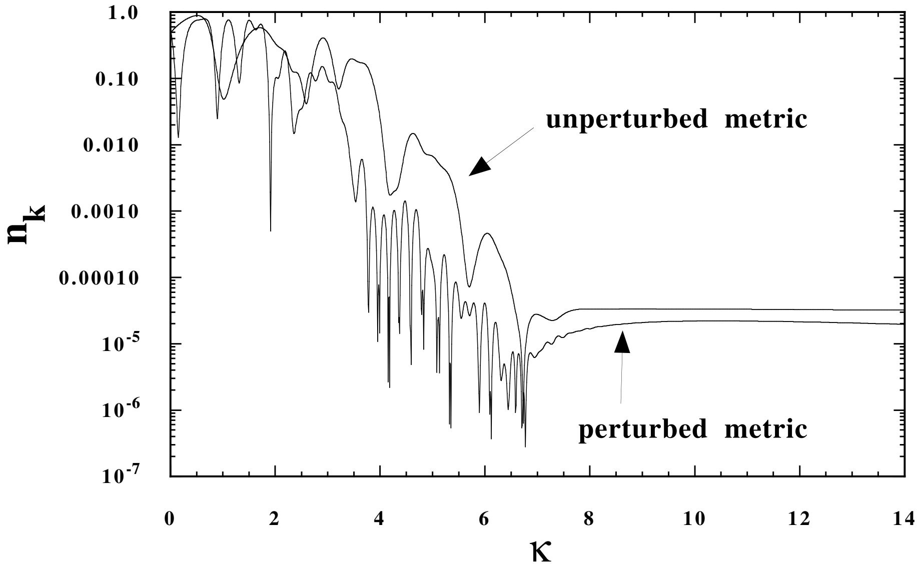

Figure 6: The spectra of the final comoving number density of fermions n_k in the perturbed and unperturbed ($\Phi_k = 0$) metric with $q_1 = 0$ and $q_2 = 10^3$ for $g^2/\lambda = 2$ and $\tilde{g} = 0$ at $x = 50$. The initial values of scalar fields are chosen as $\phi(0) = 0.5m_{\rm pl}$ and $\chi(0) = 10^{-9}m_{\rm pl}$. Metric perturbations lead to a decrease in the number of fermions produced from χ decays since they enhance $\langle\delta\phi^2\rangle$ which causes the resonance in χ to end earlier, at smaller values of $\langle\delta\chi^2\rangle$.

regime, we warn the reader that including second order metric perturbations [52] or rescattering effects [53] may change these results, because this is expected to enhance the homogeneous χ field as studied in ref. [54]. In addition to this, although we have restricted ourselves to the massless inflation, there exist other models of preheating [39] where metric perturbations will be amplified.

As long as the χ field is not strongly suppressed during inflation and gets amplified during preheating, the basic scenario of fermion production will not be changed. Nevertheless, in order to understand the effect of metric perturbations on fermion production precisely, it is required to study the dynamics of fermionic preheating including full backreaction effects in broad classes of models, which we leave to the future work.

3.2 Heavy χ field: $g^2/\lambda \gg 1$

In the case $g^2/\lambda \gg 1$, and $\tilde{g} = 0$, the homogeneous χ and the super-Hubble $\delta\chi_k$ modes are exponentially suppressed during inflation since their effective masses are much larger than the Hubble rate [54, 55]. This results in the very small values $\chi \lesssim 10^{-45}m_{\rm pl}$ at the onset of preheating. In this case, fermion production by the ψ-χ interaction is very weak, although the ϕ-ψ interaction leads to standard fermion production.

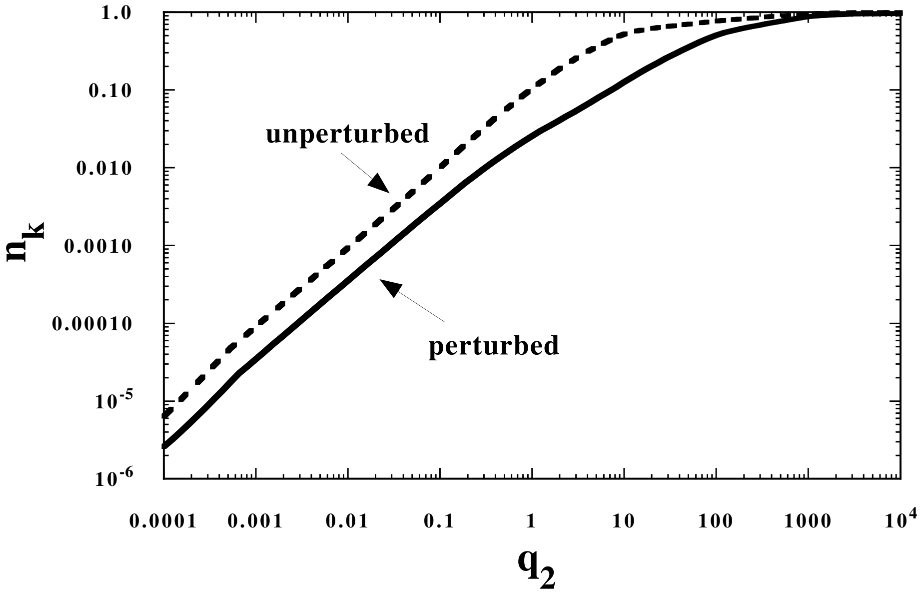

Figure 7: The maximum comoving number density of fermions n_k with $\kappa = 1$ achieved during preheating as a function of $q_2 = h_2^2/\lambda$. The parameters chosen are $g^2/\lambda = 2$, $\tilde{g} = 0$, and $q_1 = 0$ with the initial values of $\phi(0) = 0.5m_{\rm pl}$ and $\chi(0) = 10^{-9}m_{\rm pl}$. The fermion occupation number increases almost linearly with q_2 until it saturates near $n_k = 1$. The effect of the perturbed metric ($\Phi_k \neq 0$) works to reduce the maximum occupation number.

In the presence of the $\tilde{g}^2\phi^3\chi$ coupling, however, $\langle\chi\rangle \neq 0$ [39, 40]. In fact, neglecting the derivative terms in eqs. (2.9) and (2.5), one finds the following relations at the end of inflation:

$$\chi \approx -\left(\frac{\tilde{g}}{g}\right)^2 \phi\,, \qquad \delta\chi_k \approx -3\left(\frac{\tilde{g}}{g}\right)^2 \delta\phi_k\,. \tag{3.2}$$

Although these χ modes are about $(\tilde{g}/g)^2$ times smaller than those of the inflaton, they undergo parametric amplification during preheating, which leads to fermion production. As an example, we depict in figure 8 the spectra of the final number density of fermions for $g^2/\lambda = 800$ and $\tilde{g} = 10^{-3}g$. Since this case corresponds to $g^2/\lambda = 2l^2$, with $l = 20$, the χ modes close to $\kappa = 0$ are strongly enhanced. In addition to fermion production through the h_1 coupling, the ψ-χ interaction leads to an extra fermion creation as found in figure 8.

In the case of $g^2/\lambda \gg 1$ with $\tilde{g} > 0$, the super-Hubble $\delta\chi_k$ fluctuation is about $(\tilde{g}/g)^2$ times smaller than the $\delta\phi_k$ fluctuation, while the sub-Hubble $\delta\chi_k$ is the same order of $\delta\phi_k$. Since the χ-dependent source term on small scales in the r.h.s. of eq. (2.6) is generally larger than the large scale modes for $\tilde{g} \lesssim g$, the sub-Hubble metric perturbation grows faster than the the super-Hubble mode as shown in figure 9.

For $\tilde{g} \lesssim 10^{-2}g$ amplification of the $\delta\chi_k$ fluctuation terminates by backreaction effects before the super-Hubble metric perturbation begins to grow. Larger values of $\tilde{g}$ would enhance super-Hubble metric perturbations but by requiring the inflaton

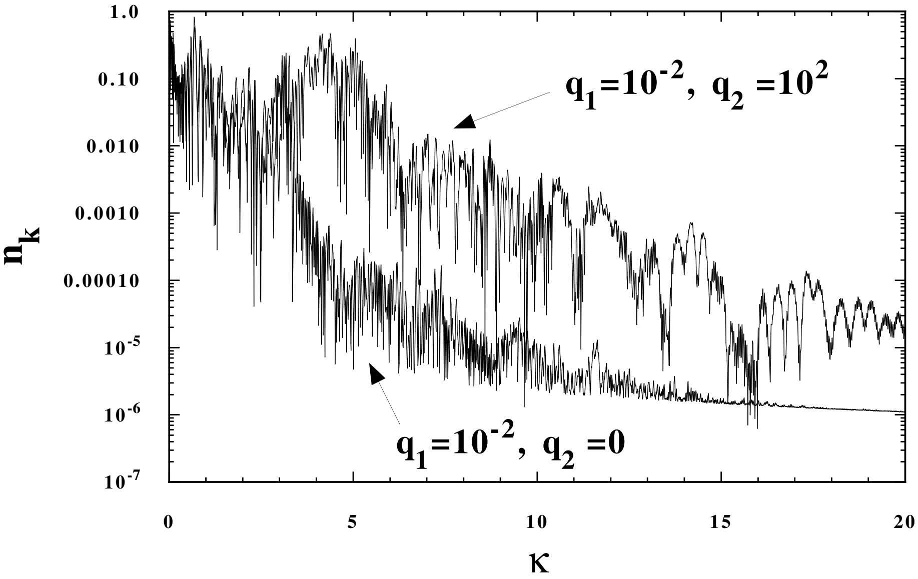

Figure 8: The final spectra of fermions n_k in the cases of $q_1 = 10^{-2}$, $q_2 = 10^2$ (top) and $q_1 = 10^{-2}$, $q_2 = 0$ (bottom) with $g^2/\lambda = 800$ and $\tilde{g} = 10^{-3}\,g$. The initial values of scalar fields are chosen as $\phi(0) = 0.5 m_{\rm pl}$ and $\chi(0) = -(\tilde{g}/g)^2\phi(0)$, values which we use for all figures in which $\tilde{g} \neq 0$, as dictated by eq. (3.2) We find that the existence of the h_2 coupling assists fermion production as in the case where $g^2/\lambda = \mathcal{O}(1)$.

to have a positive effective mass, one obtains the following relation making use of eqs. (2.8) and (3.2) :

$$\frac{\tilde{g}}{g} < \left(\frac{\lambda}{2g^2}\right)^{1/4}. \tag{3.3}$$

This means that $\tilde{g}/g$ cannot safely exceed unity[1] for $g^2/\lambda \gg 1$, which restricts the strong enhancement of super-Hubble metric perturbations, and protects the CMB from preheating. However, as long as $\tilde{g}/g$ is not much smaller than unity and the χ field escapes from the inflationary suppression, we can say that fermion production is possible only by the h_2 coupling as in the case of $g^2/\lambda = \mathcal{O}(1)$.

4. Massive fermion production

4.1 No χ decay: $h_1 \neq 0$, $h_2 = 0$

If the mass of fermion is taken into account, this generally works to suppress efficient fermion production. Let us first review the case of $h_1 \neq 0$ and $h_2 = 0$. Fermion production takes place around the region where the effective mass $m_{\rm eff} = m_\psi + h_1\phi$

[1]The main concern is that the inflaton must evolve to the correct vacuum at $\phi = 0$ rather then diverging to infinity.

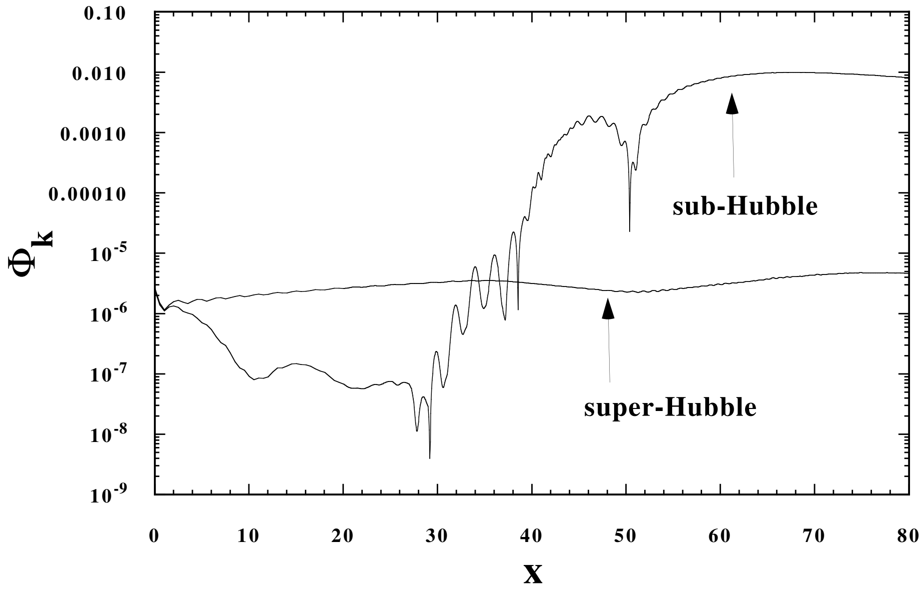

Figure 9: The evolution of metric perturbations Φ_k for the super-Hubble mode $\kappa = 10^{-26}$ and the sub-Hubble mode $\kappa = 1$ with $g^2/\lambda = 800$ and $\tilde{g} = 10^{-3}g$. The initial values of scalar fields are chosen as $\phi(0) = 0.5m_{\rm pl}$ and $\chi(0) = -(\tilde{g}/g)^2\phi(0)$. In this case, the sub-Hubble metric perturbation gets amplified, while the super-Hubble mode (and conserved quantity ζ), remain almost constant.

vanishes, which corresponds to

$$\phi_* = -\frac{m_\psi}{h_1} \,. \tag{4.1}$$

As long as the relation $m_\psi < |h_1\phi|$ is satisfied, fermions are periodically created when the inflaton passes through ϕ_*. After the amplitude of inflaton decreases under the critical value ϕ_*, fermion creation ends.

This allows an analytical estimate of the maximum mass of fermions produced through the h_1 coupling. Taking note that the conformal inflaton field and the scale factor evolve as [45, 51]

$$\tilde{\phi} = a\phi \approx \tilde{\phi}_{\rm co}\cos\left(0.8472\sqrt{\lambda}\tilde{\phi}_{\rm co}\eta\right) , \tag{4.2}$$

$$a \approx \sqrt{\frac{2\pi\lambda}{3}}\frac{\tilde{\phi}_{\rm co}^2}{m_{\rm pl}}\eta \,, \tag{4.3}$$

the effective mass of the fermion field is approximately

$$m_{\rm eff} = m_\psi + h_1\sqrt{\frac{3}{2\pi\lambda}}\frac{m_{\rm pl}}{\tilde{\phi}_{\rm co}}\frac{1}{\eta}\cos\left(0.8472\sqrt{\lambda}\tilde{\phi}_{\rm co}\eta\right) . \tag{4.4}$$

Since the minimum effective mass is achieved for $0.8472\sqrt{\lambda}\tilde{\phi}_{\rm co}\eta = \pi$, the upper limit of the fermion mass production is

$$m_\psi \lesssim \frac{h_1}{5}m_{\rm pl} \,. \tag{4.5}$$

This indicates that super-heavy fermions whose masses are of order 10^{17}-$10^{18}\,$GeV can be produced if we choose large couplings with h_1 close to unity [36, 5].

In the vicinity of $m_{\rm eff}=0$ where particles are non-adiabatically created, we can approximately express $m_{\rm eff}$ as

$$m_{\rm eff}=h_1\phi_*'(\eta-\eta_*)\,, \tag{4.6}$$

where a prime denotes the derivative with respect to η. Since the expansion rate of the universe is typically smaller than the creation rate of fermions around $m_{\rm eff}=0$, we can safely set $a(\eta_*)=1$, $a'(\eta_*)=0$ in eq. (2.17). Substituting eq. (4.6) for eq. (2.17) and introducing new variables $\tau\equiv\sqrt{h_1\phi_*'}(\eta-\eta_*)$, $p\equiv k/\sqrt{h_1\phi_*'}$, $d\tau\equiv\sqrt{h_1\phi_*'}d\eta$, one obtains [5, 36]

$$\left[\frac{d^2}{d\tau^2}+p^2+\tau^2-i\right]\tilde{\psi}_k=0\,. \tag{4.7}$$

The solution of this equation is described by the parabolic cylinder function, and the comoving number density of fermions is analytically derived as [5]

$$n_k\approx\exp\left(-\frac{\pi^2k^2}{\lambda|\phi_*'|}\right)=\exp\left[-\frac{\pi\phi(0)}{|d\phi_*/dx|}\frac{\kappa^2}{\sqrt{q_1}}\right]. \tag{4.8}$$

This relation means that larger values of q_1 and $|d\phi_*/dx|$ lead to the production of massive particles with higher momenta. Massive fermions can be instantly created even after one oscillation of inflaton. Their occupation number increases to of order unity for low momentum modes $\kappa\sim 0$. Note that if the coupling h_1 satisfies the relation $m_\psi>|h_1\phi|$ at the beginning of preheating, the spectrum of eq. (4.8) can not be applied.

In order to confirm above analytic estimates, we have numerically studied the evolution of the comoving number density of fermions. In figure 10, we plot the evolution of n_k for a bare mass $m_\psi=10^{-5}m_{\rm pl}\sim 10^{14}\,$GeV with $g^2/\lambda=2$, $\kappa=1$, and $q_1=10^3,10^4,10^5$.

In this case, the analytic estimate of eq. (4.5) predicts that the coupling h_1 is required to be $h_1\gtrsim 5\times10^{-5}$ for efficient fermion production, which corresponds to $q_1\gtrsim 2.5\times10^3$ with $\lambda=10^{-12}$. This is elegantly confirmed in in figure 10 where fermions are not non-adiabatically created for the case of $q_1=10^3$. In this case, since the bare mass m_ψ always surpasses the $|h_1\phi|$ term during the whole stage of preheating, we can not expect stochastic increase of n_k.

On the other hand, for $q_1=10^4$ and 10^5, we find that the comoving number density increases to order unity when the inflaton passes through ϕ_*. For $q_1=10^4$, non-adiabatic amplification of fermions occurs only once around $x\approx 3$, after which the amplitude of the inflaton decreases below the critical threshold $|\phi_*|$. When $q_1=10^5$, in contrast, n_k reaches unity several times. If one chooses larger values of q_1, non-adiabatic creation of fermions takes place periodically as long as the amplitude of inflaton is larger than $|\phi_*|$.

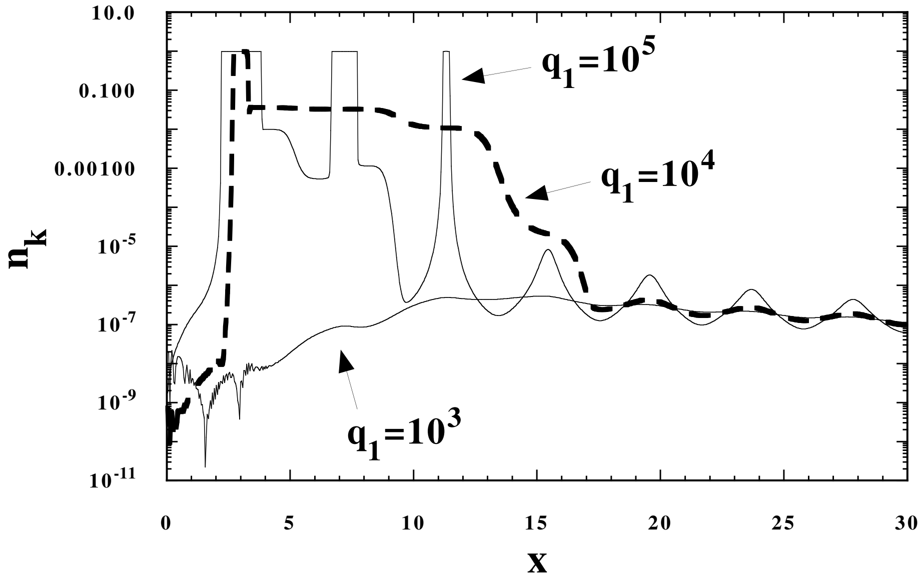

Figure 10: The time evolution of the comoving number density of fermions n_k with mass $m_\psi = 10^{-5} m_{\rm pl}$ and the coupling $q_1 = 10^3, 10^4, 10^5$ with $q_2 = 0$, $g^2/\lambda = 2$, and $\tilde{g} = 0$ for the sub-Hubble mode $\kappa = 1$. The initial values of the scalar fields are $\phi(0) = 0.5 m_{\rm pl}$ and $\chi(0) = 10^{-9} m_{\rm pl}$.

In figure 11, we depict the spectrum n_k after one oscillation of the inflaton with bare fermion mass $m_\psi = 10^{-5} m_{\rm pl}$ and couplings $q_1 = 10^3, 10^4, 10^5$ with $q_2 = 0$, $g^2/\lambda = 2$, and $\tilde{g} = 0$. When $q_1 = 10^4$ and 10^5 where fermions are created around $x \approx 3$, the phase space density of produced particle is described by eq. (4.8). The larger coupling h_1 leads to the enhancement of higher momentum modes, which we can confirm in figure 11.

Since non-adiabatic fermion production does not occur for $q_1 \lesssim 2.5 \times 10^3$ in this case, the spectrum for $q_1 = 10^3$ does not obey the analytic estimate of eq. (4.8) (see the inset of figure 11). Note that the fermion spectra change with the passage of time. However, for a short period where fermions are created in the vicinity of $m_{\rm eff} = 0$, the spectrum is well described by eq. (4.8).

Super-heavy fermions with masses greater than the GUT scale $m_\psi \sim 10^{16}\,$GeV can be produced if the Yukawa-coupling is larger than $h_1 \approx 5 \times 10^{-3}$ by the same mechanism described above. These massive fermions might have played an important role in leptogenesis scenario or be candidates of the super-heavy dark matter [34].

4.2 Instant preheating considered: $h_1 = 0$, $h_2 \neq 0$

Let us next consider the case with $h_1 = 0$ and $h_2 \neq 0$. When $1 < g^2/\lambda < 3$, since long-wavelength modes of the χ field are resonantly amplified, one might assume that massive fermions are efficiently produced from the beginning of preheating. However, for instant production of super-massive fermions, the whole $h_2\chi$ term in eq. (1.2) must be comparable to the bare mass m_ψ at the start of preheating.

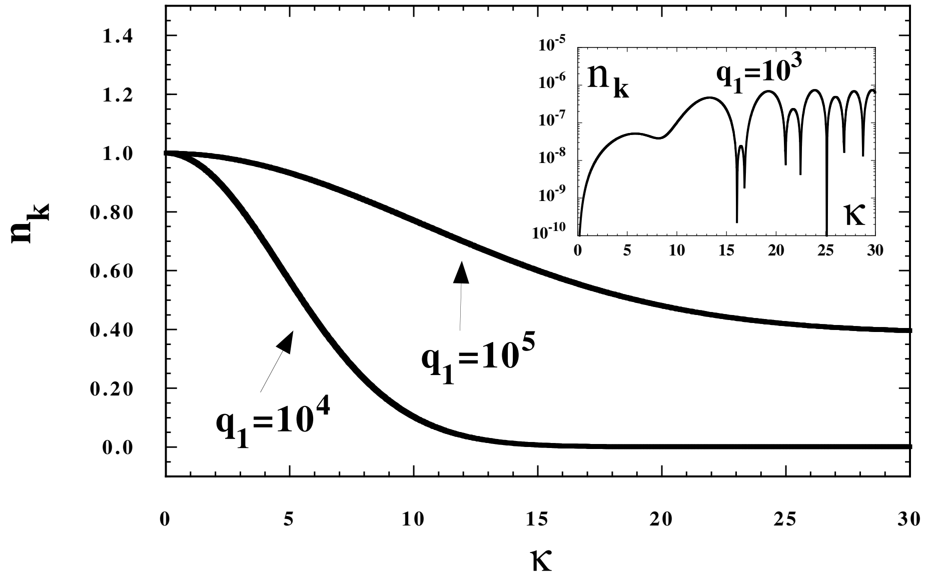

Figure 11: The spectra of the comoving number density of fermions n_k after one oscillation of the inflaton with bare fermion mass $m_\psi = 10^{-5} m_{\rm pl}$ and $g_2 = 0$. For this mass, simple analytical estimates imply a threshold for instant production at around $q_1 = 2.5 \times 10^3$. The main figure shows n_k for $q_1 = 10^4, 10^5$ with $g^2/\lambda = 2$, and $\tilde{g} = 0$. The spectra match the simple analytical result of eq. (4.8) very well. **Inset**: n_k vs κ for q_1 below the threshold, $q_1 = 10^3$. Essentially no instant production of fermions takes place. The initial values of scalar fields are chosen to be $\phi(0) = 0.5 m_{\rm pl}$ and $\chi(0) = 10^{-9} m_{\rm pl}$.

This condition is rather difficult to achieve unless both the coupling h_2 and the initial value of χ are large. For example, for an initial value $\chi(0) = 10^{-9} m_{\rm pl}$ which we used in the simulations of figures 2–7, even strong couplings $h_2 \sim 1$ yield $h_2 \chi(0) \sim 10^{-9} m_{\rm pl}$. This indicates that super-massive fermions heavier than $m_\psi \sim 10^{-6} m_{\rm pl}$ cannot be produced at the initial stage. In this case however, once the amplitude of the χ field grows through parametric resonance and the $|h_2 \chi|$ becomes larger than m_ψ, χ-ψ production begins to be relevant.

If $g^2/\lambda = \mathcal{O}(1)$ and $\tilde{g} = 0$, large initial χ values $\chi(0) \gg 10^{-9} m_{\rm pl}$ will lead to the strong amplification of super-Hubble metric perturbations. In order that the model not conflict with observations one requires $\chi(0) < 10^{-9} m_{\rm pl}$ which means that instant fermion production of massive fermions is rather difficult unless h_2 is unnaturally large.

When $g^2/\lambda \gg 1$ with the $\tilde{g}$ coupling, the χ field is constrained by eq. (3.2). Since the super-Hubble $\delta\chi_k$ fluctuation is about $(\tilde{g}/g)^2$ times smaller relative to $\delta\phi_k$, even the initial value of $\chi(0) \sim 10^{-6} m_{\rm pl}$ does not result in the strong enhancement of super-Hubble metric perturbations as shown in figure 9. Making use of the relation (3.2), the effective mass of fermions at the end of inflation is approximately given by

$$m_{\rm eff} \approx m_\psi - h_2 \left(\frac{\tilde{g}}{g}\right)^2 \phi(0)\,. \tag{4.9}$$

When h_2 and $\phi(0)$ are positive, $m_{\rm eff}$ can take negative values at the end of inflation for large couplings h_2 and ratio $\tilde{g}/g$. Since the amplitude of the χ field decreases at the initial stage of preheating due to cosmic expansion, instant fermion production takes place when $m_{\rm eff}$ vanishes before the χ fluctuation is amplified through resonance. This case corresponds to

$$h_2 \left(\frac{\tilde{g}}{g}\right)^2 \gtrsim 2\frac{m_\psi}{m_{\rm pl}}\,, \tag{4.10}$$

where we used the value $\phi(0) = 0.5 m_{\rm pl}$.

For example, for a coupling of $h_2 = 1$ and $\tilde{g}/g = 10^{-2}$, fermions with bare masses of order $m_\psi \sim 5\times 10^{-5} m_{\rm pl} \sim 5\times 10^{14}\,{\rm GeV}$ are produced at the initial stage. For instant creation of GUT scale fermions $m_\psi \sim 10^{16}\,{\rm GeV}$, we require $h_2(\tilde{g}/g)^2 \gtrsim 10^{-3}$. Due to the existence of the $\tilde{g}/g$ term, which is required to be smaller than unity for successful inflation, massive fermion production via the h_2 coupling is generally hard to realize relative to the h_1 case [c.f. eq. (4.5)].

It is important to recognise that eq. (4.10) is the condition of instant fermion production at the initial stage of preheating. Even when $h_2\chi(0) \ll m_\psi$ initially, subsequent exponential growth of χ leads to fermion production at the later stages of preheating.

These effects are illustrated in figure 12, where we show the evolution of n_k for fermions with mass $m_\psi = 10^{-6} m_{\rm pl}$ and coupling $g^2/\lambda = 5.0\times 10^3$, $\tilde{g}/g = 3.0\times 10^{-3}$, $h_2 = 1$ and 0.1 for the mode $\kappa = 2$. The condition of eq. (4.10) is satisfied for $h_2 = 1$, which leads to instant fermion production found in figure 12. Since the expansion of the universe reduces the amplitude of the χ field at the first stage of preheating, fermionic preheating is frozen for some time once $m_{\rm eff}$ vanishes at $x \sim 2$. For low momentum modes $\kappa \sim 0$, n_k increases to unity by instant fermion production.

For $x \gtrsim 30$, parametric amplification of the χ field causes non-adiabatic fermion production (see figure 12). For $h_2 = 0.1$, where the condition of eq. (4.10) is not satisfied, n_k does not increase at the initial stage. However, n_k begins to grow after the χ field has been sufficiently amplified. Since the homogeneous χ field periodically changes between positive and negative values, we find that fermion productions occurs in the same way even for the case of $h_2 < 0$ at the final stage of preheating.

In figure 13, we show the maximum number density of fermions as a function of $\tilde{g}/g$ in two cases of $m_\psi = 10^{-6} m_{\rm pl}$ and $m_\psi = 10^{-8} m_{\rm pl}$ with $g^2/\lambda = 5\times 10^3$, $h_1 = 0$, and $h_2 = 1$. We find that large values of $\tilde{g}/g$ are needed as one increases the fermion bare mass. In order to produce GUT scale fermions $m_\psi \sim 10^{16}\,{\rm GeV}$, we require strong coupling $\tilde{g}/g$ close to its upper bound of eq. (3.3).

4.3 Both couplings taken into account: $h_1 \neq 0$, $h_2 \neq 0$

If both h_1 and h_2 are non zero, fermionic preheating strongly depends both upon the absolute and relative strengths of the couplings. In the presence of the $\tilde{g}$ coupling, the

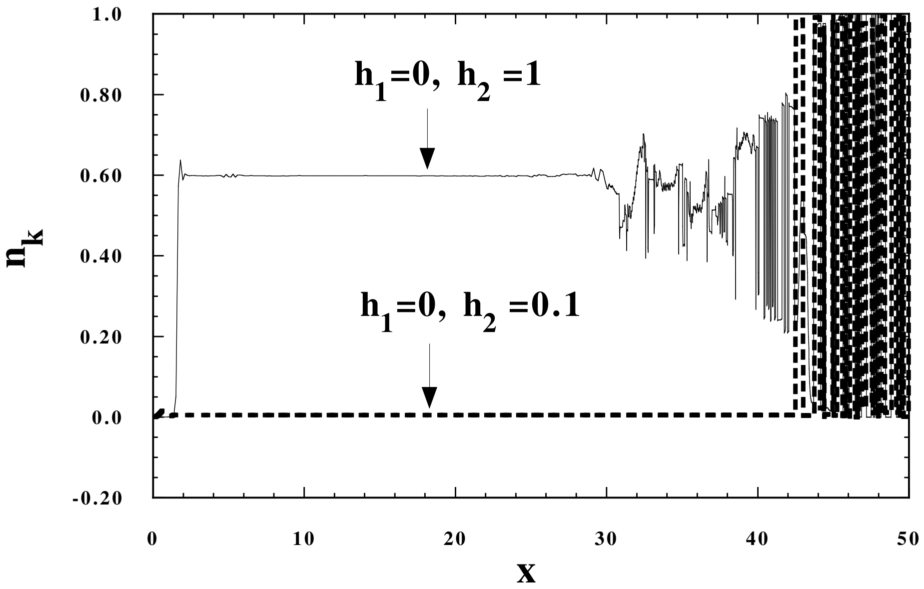

Figure 12: Instant production followed by chaotic production of fermions. The evolution of the comoving number density of fermions n_k with bare mass $m_\psi = 10^{-6} m_{\rm pl}$ and the coupling $h_2 = 1$ (solid), $h_2 = 0.1$ (dotted) with $h_1 = 0$, $g^2/\lambda = 5.0 \times 10^3$, and $\tilde{g}/g = 3.0 \times 10^{-3}$ for the mode $\kappa = 2$. The initial conditions are $\phi(0) = 0.5 m_{\rm pl}$ and $\chi(0) = -(\tilde{g}/g)^2 \phi(0)$.

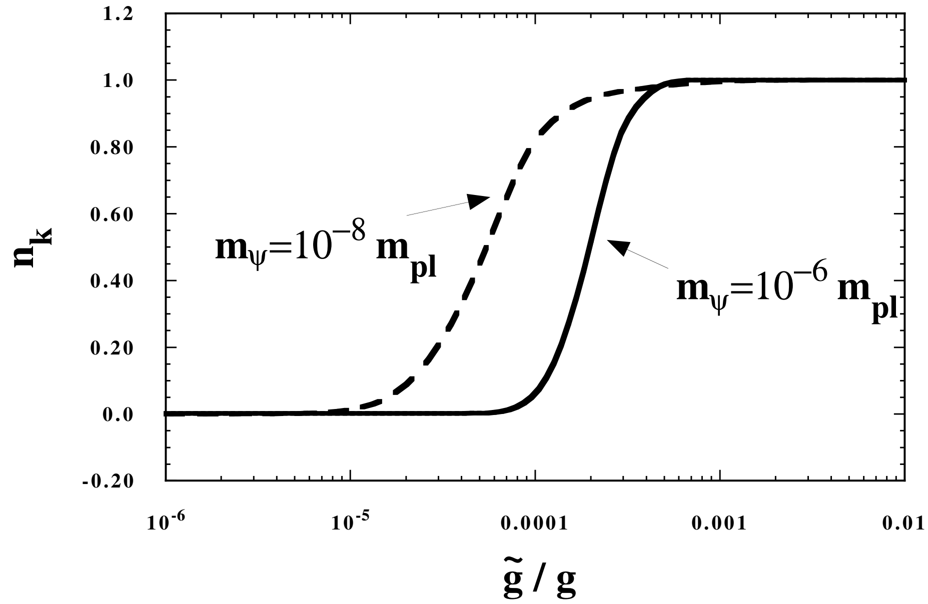

Figure 13: The maximum comoving number density of fermions n_k for the Hubble-scale mode $\kappa = 1$ as a function of $\tilde{g}/g$ for $g^2/\lambda = 5.0 \times 10^3$, $h_1 = 0$, and $h_2 = 1$ with mass $m_\psi = 10^{-6} m_{\rm pl}$ and $m_\psi = 10^{-8} m_{\rm pl}$. As the fermion mass increases, large $\tilde{g}/g$ is required before $\chi \to \psi$ decays become efficient.

effective mass of fermion at the initial stage of preheating is approximately given by

$$m_{\rm eff} \approx m_\psi + \left[h_1 - h_2 \left(\frac{\tilde{g}}{g} \right)^2 \right] \phi \,. \tag{4.11}$$

JHEP08(2000)019

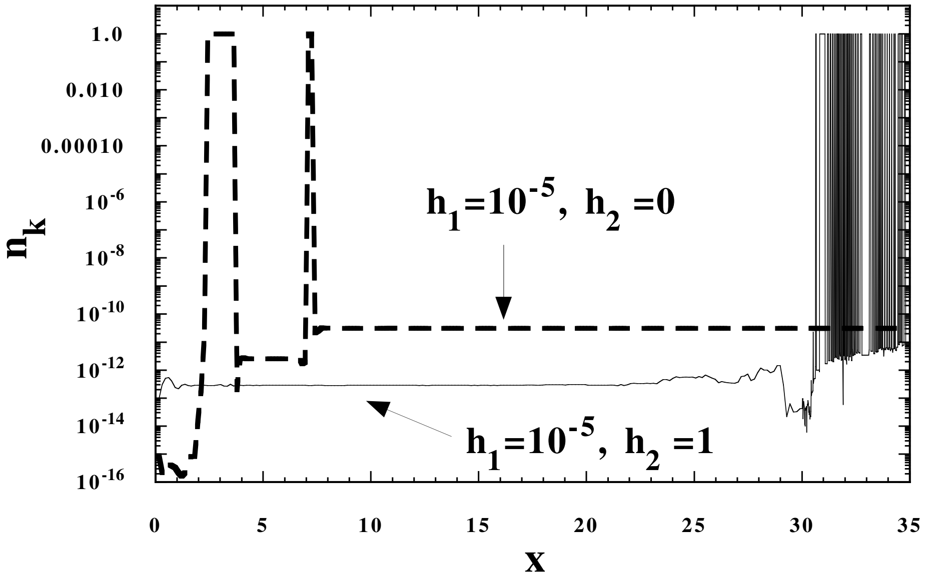

Figure 14: h_2-suppression of instant fermion production. Number density of fermions n_k vs x in the cases $h_1 = 10^{-5}$, $h_2 = 1$ (solid), and $h_1 = 10^{-5}$, $h_2 = 0$ (dotted), with $g^2/\lambda = 5.0 \times 10^3$, $\tilde{g}/g = 3.0 \times 10^{-3}$, and $m_\psi = 5.0 \times 10^{-7} m_{\rm pl}$ for the mode $\kappa = 1$. The initial conditions chosen were $\phi(0) = 0.5 m_{\rm pl}$ and $\chi(0) = -(\tilde{g}/g)^2\phi(0)$. The h_2 coupling reduces the violation of the adiabatic condition and removes the initial instant production present when $h_2 = 0$ for an otherwise equivalent parameter set.

This relation indicates that the negative coupling h_2 will assist fermion production caused by the positive h_1 coupling. Conversely, when h_1 and h_2 are both positive, including the h_2 coupling generally works as to *suppress* fermion production as long as $h_1 \geq h_2(\tilde{g}/g)^2$.

Let us consider concrete cases for the couplings: $h_1 = 10^{-5}$, $h_2 = 1$, $g^2/\lambda = 5.0 \times 10^3$, and $\tilde{g}/g = 3.0 \times 10^{-3}$ with $m_\psi = 5.0 \times 10^{-7} m_{\rm pl}$. When $h_2 = 0$, fermions are stochastically produced at the initial phase of preheating when the inflaton passes causes $m_{\rm eff} = 0$.

However, the h_2 coupling makes the second term in eq. (4.11) smaller than the bare mass m_ψ. Hence instant fermion production does not take place in this case, although n_k increases for $x \gtrsim 30$ due to the resonance from the h_2 coupling (see figure 14).

This suppression is relevant for large coupling h_2 relative to h_1. When h_1 and h_2 are of the same order, fermion production mainly proceeds by the h_1 coupling, because the ratio $\tilde{g}/g$ is typically smaller than unity.

When $h_1 = 0$ and $h_2 \neq 0$, massive fermions which satisfy the relation (4.10) are instantly produced. This typically requires rather large coupling h_2 for producing massive fermions whose masses are greater than $m_\psi \sim 10^{-6} m_{\rm pl}$. However, introducing the h_1 coupling even much smaller than h_2 can potentially alter the dynamics of fermion production.

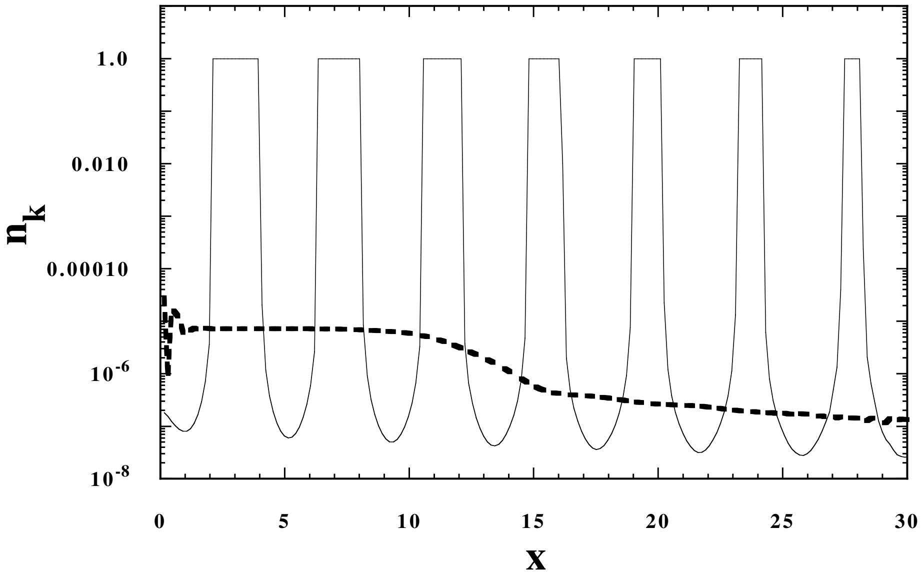

Figure 15: Significant instant production. The time evolution of fermion number density n_k for $h_1 = 10^{-3}$, $h_2 = 1$ (solid) and $h_1 = 0$, $h_2 = 1$ (dotted) with $g^2/\lambda = 5.0 \times 10^3$, $\tilde{g}/g = 3.0 \times 10^{-3}$, and $m_\psi = 10^{-5} m_{\rm pl}$ for the mode $\kappa = 1$. The initial scalar field amplitudes are $\phi(0) = 0.5 m_{\rm pl}$ and $\chi(0) = -(\tilde{g}/g)^2 \phi(0)$.

For example, in the case of $h_1 = 0$, $h_2 = 1$, $g^2/\lambda = 5.0 \times 10^3$, and $\tilde{g}/g = 3.0 \times 10^{-3}$ with $m_\psi = 10^{-5} m_{\rm pl}$, we can not expect stochastic amplification of fermions at the initial stage while n_k increases for $x \gtrsim 40$ after the χ field is sufficiently produced. Including the h_1 coupling greater than 10^{-4}, fermions are periodically created from the beginning of preheating (see figure 15). This means that the instant preheating scenario discussed in ref. [27] can be further strengthened by taking into account the h_1 coupling.

5. Concluding remarks and discussion

Production of gravitinos and other fermions after inflation has become an issue of great importance in modern cosmology since their creation provides a window into the early universe and provides a powerful mechanism for distinguishing between different models and regions of parameter space. As such the phenomenon of fermion creation deserves to be studied in great detail, especially as unusual and non-trivial results are common as the complexity of the model scenario is increased.

With this motivation we have studied spin-1/2 fermion production over a 6-dimensional coupling constant parameter space with both and without metric perturbations included. This parameter space spans both the direct resonant decay of the inflaton into both massive and massless fermions and the indirect $\phi \to \chi \to \psi$ decays of instant preheating.

Previous studies have limited themselves to 3 or 4-dimensional parameter spaces in which either there is only the ϕ-ψ system [30, 32, 33] or in which the χ field has zero vacuum expectation value during inflation and there is no ϕ-ψ coupling [27].

We have verified the general expectation that increasing the ϕ-ψ (h_1) and χ-ψ (h_2) couplings typically leads to enhanced fermion production though with an interesting exception. In the case where $\langle\chi\rangle > 0$ and both h_1 and h_2 are non negative, increasing h_2 leads to a *decrease* in the production of fermions due to a weakening of the degree of violation of the adiabatic condition, c.f. eq. (4.11).

Further, if $\langle\chi\rangle = 0$ during inflation (i.e. $\tilde{g} = 0$) instant preheating is initially sub-dominant relative to direct decays of the inflaton (figure 3) but later becomes important due to resonant growth of the χ field. If $\langle\chi\rangle \neq 0$ during inflation an initial burst of instant preheating is typically followed by a quiescent period and then a sudden chaotic burst of fermion production when the χ field becomes large due to resonance (see figures 12 and 14).

In addition we found that metric perturbations had an important impact on n_k at weak coupling (small h_1, h_2) while their effect is weak at strong coupling due to the saturation caused by the Pauli exclusion principle. In the cases we studied (figures 6 and 7) the effect of metric perturbations was to *reduce* the production of fermions. This can be understood as due to the enhancement of the inflaton resonance and hence inflaton variance $\langle\delta\phi^2\rangle$ when metric perturbations are included [44, 47]. This causes backreaction to end the resonance earlier, before the inflaton and χ fields have decayed significantly to fermions.

Fermion production via the h_2 coupling is sensitive to the value of χ at the end of inflation. Although we considered the massless inflaton model where the χ field may escape suppression during inflation for $g^2/\lambda = \mathcal{O}(1)$, there exist other scenarios of inflation in which the exponential suppression can be avoided. One such scenario is a non-minimally coupled χ field [56, 47].

During inflation, the homogeneous and super-Hubble χ modes exhibit exponential growth for negative non-minimal coupling ξ as pointed out in ref. [57], which may drastically change the scenario of instant fermionic preheating described in this paper. This amplification of the χ field is expected to work even for small values of $|\xi|$ smaller than unity in a model-independent way. Nevertheless this requires a detailed study including the evolution of metric perturbations during inflation and preheating, which we leave to future work.

Acknowledgments

We thank Jürgen Baacke, Mar Bastero-Gil, Christopher Gordon, David Kaiser, Kei-ichi Maeda, Takashi Torii and the referee for useful comments. ST was supported partially by a Grant-in-Aid for Scientific Research Fund of the Ministry of Education, Science and Culture (No. 09410217 and Specially Promoted Research No. 08102010), by the Waseda University Grant for Special Research Projects. FV is supported by CONYCIT scholarship Ref:115625/116522.

References

[1] N.D. Birrel and P.C.W. Davies, *Quantum fields in curved space*, 1980.

[2] C. Velo and D. Zwanziger, *Propagation and quantization of Rarita-Schwinger waves in an external electromagnetic potential*, *Phys. Rev.* **186** (1969) 1337.

[3] S. Deser and B. Zumino, *Broken supersymmetry and supergravity*, *Phys. Rev. Lett.* **38** (1977) 1433.

[4] A. Berera and T.W. Kephart, *Ubiquitous inflaton in string-inspired models*, *Phys. Rev. Lett.* **83** (1999) 1084 [hep-ph/9904410].

[5] D.J. H. Chung, E.W. Kolb, A. Riotto and I.I. Tkachev, *Probing planckian physics: resonant production of particles during inflation and features in the primordial power spectrum*, *Phys. Rev.* **D 62** (2000) 043508 [hep-ph/9910437].

[6] T. Moroi, *Effects of the gravitino on the inflationary universe*, hep-ph/9503210;
S. Sarkar, *Big-Bang nucleosynthesis and physics beyond the standard model*, *Rept. Prog. Phys.* **59** (1996) 1493 [hep-ph/9602260];
R. Kallosh, L. Kofman, A. Linde and A.V. Proeyen, *Superconformal symmetry, supergravity and cosmology*, hep-th/0006179.

[7] T. Asaka, K. Hamaguchi, M. Kawasaki and T. Yanagida, *Leptogenesis in inflaton decay*, *Phys. Lett.* **B 464** (1999) 12 [hep-ph/9906366]; *Leptogenesis in inflationary universe*, *Phys. Rev.* **D 61** (2000) 083512 [hep-ph/9907559].

[8] M. Birkel and S. Sarkar, *Extremely high-energy cosmic rays from relic particle decays*, *Astropart. Phys.* **9** (1998) 297 [hep-ph/9804285].

[9] V. Kuzmin and I.I. Tkachev, *Ultrahigh-energy cosmic rays, superheavy long living particles, and matter creation after inflation*, *Sov. Phys. JETP Lett.* **68** (1998) 271 [hep-ph/9802304];
D.J.H. Chung, E.W. Kolb and A. Riotto, *Superheavy dark matter*, *Phys. Rev.* **D 59** (1999) 023501 [hep-ph/9802238].

[10] A.D. Dolgov and A.D. Linde, *Baryon asymmetry in inflationary universe*, *Phys. Lett.* **B 116** (1982) 329;
L.F. Abbott, E. Farhi and M.B. Wise, *Particle production in the new inflationary cosmology*, *Phys. Lett.* **B 117** (1982) 29.

[11] J. Ellis, A.D. Linde and D.V. Nanopoulos, *Inflation can save the gravitino*, *Phys. Lett.* **B 118** (1982) 59;

M. Kawasaki and T. Moroi, *Gravitino production in the inflationary universe and the effects on Big Bang nucleosynthesis*, *Prog. Theor. Phys.* **93** (1995) 879 [hep-ph/9403364].

[12] H. Fujisaki, K. Kumekawa, M. Yamaguchi and M. Yoshimura, *Particle production and gravitino abundance after inflation*, *Phys. Rev.* **D 54** (1996) 2494 [hep-ph/9511381].

[13] J.H. Traschen and R.H. Brandenberger, *Particle production during out-of-equilibrium phase transitions*, *Phys. Rev.* **D 42** (1990) 2491.

[14] L. Kofman, A. Linde and A.A. Starobinsky, *Reheating after inflation*, *Phys. Rev. Lett.* **73** (1994) 3195 [hep-th/9405187];
Y. Shtanov, J. Traschen and R. Brandenberger, *Universe reheating after inflation*, *Phys. Rev.* **D 51** (1995) 5438 [hep-ph/9407247].

[15] L. Kofman, A. Linde and A.A. Starobinsky, *Towards the theory of reheating after inflation*, *Phys. Rev.* **D 56** (1997) 3258 [hep-ph/9704452].

[16] D. Boyanovsky, H.J. de Vega, R. Holman, and J.F.J. Salgado, *Analytic and numerical study of preheating dynamics*, *Phys. Rev.* **D 54** (1996) 7570.

[17] A.L. Maroto and A. Mazumdar, *Production of spin 3/2 particles from vacuum fluctuations*, *Phys. Rev. Lett.* **84** (2000) 1655 [hep-ph/9904206].

[18] R. Kallosh, L. Kofman, A. Linde and A.V. Proeyen, *Gravitino production after inflation*, *Phys. Rev.* **D 61** (2000) 103503 [hep-th/9907124].

[19] G.F. Giudice, I. Tkachev and A. Riotto, *Non-thermal production of dangerous relics in the early universe*, *J. High Energy Phys.* **08** (1999) 009 [hep-ph/9907510].

[20] M. Lemoine, *Gravitational production of gravitinos*, *Phys. Rev.* **D 60** (1999) 103522 [hep-ph/9908333].

[21] G.F. Giudice, A. Riotto and I. Tkachev, *Thermal and non-thermal production of gravitinos in the early universe*, *J. High Energy Phys.* **11** (1999) 036 [hep-ph/9911302].

[22] A.L. Maroto and J.R. Pelaez, *The equivalence theorem and the production of gravitinos after inflation*, *Phys. Rev.* **D 62** (2000) 023518 [hep-ph/9912212].

[23] M. Bastero-Gil and A. Mazumdar, *Gravitino production in hybrid inflationary models*, hep-ph/0002004.

[24] A.D. Dolgov and D.P. Kirilova, *On particle creation by a time dependent scalar field*, *Sov. J. Nucl. Phys.* **51** (1990) 172.

[25] B.A. Bassett, *Inflationary reheating classes via spectral methods*, *Phys. Rev.* **D 58** (1998) 021303 [hep-ph/9709443].

[26] B.A. Bassett and F. Tamburini, *Inflationary reheating in grand unified theories*, *Phys. Rev. Lett.* **81** (1998) 2630 [hep-ph/9804453].

[27] G. Felder, L. Kofman and A. Linde, *Instant preheating*, *Phys. Rev.* **D 59** (1999) 123523 [hep-ph/9812289].

[28] D. Boyanovsky, M. D'Attanasio, H.J. de Vega, R. Holman and D.S. Lee, *Reheating and thermalization: linear versus nonlinear relaxation*, *Phys. Rev.* **D 52** (1995) 6805 [hep-ph/9507414].

[29] D. Boyanovsky, H.J. de Vega, D.S. Lee, Y.J. Ng and S.Y. Wang, *Fermion damping in a fermion-scalar plasma*, *Phys. Rev.* **D 59** (1999) 105001 [hep-ph/9810393].

[30] J. Baacke, K. Heitmann and C. Patzold, *Nonequilibrium dynamics of fermions in a spatially homogeneous scalar background field*, *Phys. Rev.* **D 58** (1998) 125013 [hep-ph/9806205].

[31] J. Baacke and C. Patzold, *Renormalization of the nonequilibrium dynamics of fermions in a flat FRW universe*, hep-ph/9912505.

[32] S.A. Ramsey, B.L. Hu and A.M. Stylianopoulos, *Nonequilibrium inflaton dynamics and reheating, 2. Fermion production, noise and stochasticity*, *Phys. Rev.* **D 57** (1998) 6003 [hep-ph/9709267].

[33] P.B. Greene and L. Kofman, *Preheating of fermions*, *Phys. Lett.* **B 448** (1999) 6 [hep-ph/9807339].

[34] G.F. Giudice, M. Peloso, A. Riotto and I. Tkachev, *Production of massive fermions at preheating and leptogenesis*, *J. High Energy Phys.* **08** (1999) 014 [hep-ph/9905242].

[35] J. Garcia-Bellido, S. Mollerach, and E. Roulet, *J. High Energy Phys.* **02** (2000) 034.

[36] P.B. Greene and L. Kofman, *On the theory of fermionic preheating*, hep-ph/0003018.

[37] M. Peloso and L. Sorbo, *Preheating of massive fermions after inflation: analytical results*, *J. High Energy Phys.* **05** (2000) 016 [hep-ph/0003045].

[38] O. Bertolami and G.G. Ross, *One scale supersymmetric inflationary models*, *Phys. Lett.* **B 171** (1986) 46;
G.G. Ross and S. Sarkar, *Successful supersymmetric inflation*, *Nucl. Phys.* **B 461** (1996) 597 [hep-ph/9506283].

[39] B.A. Bassett, C. Gordon, R. Maartens and D.I. Kaiser, *Restoring the sting to metric preheating*, *Phys. Rev.* **D 61** (2000) 061302 [hep-ph/9909482].

[40] S. Tsujikawa, *Preheating with extra dimensions*, *J. High Energy Phys.* **07** (2000) 024 [hep-ph/0005105].

[41] V. F. Mukhanov, H. A. Feldman, and R. H. Brandenberger, *Phys. Rep.* **215** (1992) 293.

[42] H. Kodama and T. Hamazaki, *Evolution of cosmological perturbations in a stage dominated by an oscillatory scalar field*, *Prog. Theor. Phys.* **96** (1996) 949 [gr-qc/9608022];
Y. Nambu and A. Taruya, *Evolution of cosmological perturbation in reheating phase of the universe*, *Prog. Theor. Phys.* **97** (1997) 83 [gr-qc/9609029];
F. Finelli and R. Brandenberger, *Parametric amplification of gravitational fluctuations during reheating*, *Phys. Rev. Lett.* **82** (1999) 1362 [hep-ph/9809490];
M. Parry and R. Easther, *Preheating and the Einstein field equations*, *Phys. Rev.* **D 59** (1999) 061301 [hep-ph/9809574].

[43] B.A. Bassett, D.I. Kaiser and R. Maartens, *General relativistic preheating after inflation*, *Phys. Lett.* **B 455** (1999) 84 [hep-ph/9808404].

[44] B.A. Bassett, F. Tamburini, D.I. Kaiser and R. Maartens, *Metric preheating and limitations of linearized gravity, 2*, *Nucl. Phys.* **B 561** (1999) 188 [hep-ph/9901319].

[45] P.B. Greene, L. Kofman, A. Linde and A.A. Starobinsky, *Structure of resonance in preheating after inflation*, *Phys. Rev.* **D 56** (1997) 6175 [hep-ph/9705347];
D.I. Kaiser, *Preheating in an expanding universe: analytic results for the massless case*, *Phys. Rev.* **D 56** (1997) 706 [hep-ph/9702244];
D. Boyanovsky, C. Destri, H.J. de Vega, R. Holman and J. Salgado, *Asymptotic dynamics in scalar field theory: anomalous relaxation*, *Phys. Rev.* **D 57** (1998) 7388 [hep-ph/9711384].

[46] D. Boyanovsky et al., *Scalar field dynamics in Friedman-Robertson-Walker spacetimes*, *Phys. Rev.* **D 56** (1997) 1939 [hep-ph/9703327].

[47] B.A. Bassett and F. Viniegra, *Massless metric preheating*, *Phys. Rev.* **D 62** (2000) 043507 [hep-ph/9909353].

[48] F. Finelli and R. Brandenberger, *Parametric amplification of metric fluctuations during reheating in two field models*, hep-ph/0003172.

[49] D. Wands, K.A. Malik, D.H. Lyth and A.R. Liddle, *A new approach to the evolution of cosmological perturbations on large scales*, *Phys. Rev.* **D 62** (2000) 043527 [astro-ph/0003278].

[50] S.Y. Khlebnikov and I.I. Tkachev, *Classical decay of inflaton*, *Phys. Rev. Lett.* **77** (1996) 219 [hep-ph/9603378].

[51] S. Tsujikawa, K. Maeda and T. Torii, *Preheating of the nonminimally coupled inflaton field*, *Phys. Rev.* **D 61** (2000) 103501 [hep-ph/9910214];
S. Tsujikawa and B.A. Bassett, *A new twist to preheating*, *Phys. Rev.* **D 62** (2000) 043510 [hep-ph/0003068].

[52] L.R. W. Abramo, R.H. Brandenberger and V.F. Mukhanov, *The energy-momentum tensor for cosmological perturbations*, *Phys. Rev.* **D 56** (1997) 3248 [gr-qc/9704037].

JHEP08(2000)019

[53] R. Easther and M. Parry, *Gravity, parametric resonance and chaotic inflation*, hep-ph/9910441;
M. Parry, *Topics in inflationary preheating*, PhD Thesis.

[54] K. Jedamzik and G. Sigl, *On metric preheating*, *Phys. Rev.* **D 61** (2000) 023519 [hep-ph/9906287].

[55] P. Ivanov, *On generation of metric perturbations during preheating*, *Phys. Rev.* **D 61** (2000) 023505 [astro-ph/9906415];
A.R. Liddle, D.H. Lyth, K.A. Malik and D. Wands, *Super-horizon perturbations and preheating*, *Phys. Rev.* **D 61** (2000) 103509 (R) [hep-ph/9912473];
A.B. Henriques and R.G. Moorhouse, *Post-inflationary reheating*, *Phys. Rev.* **D 62** (2000) 063512 [hep-ph/0003141].

[56] B.A. Bassett and S. Liberati, *Geometric reheating after inflation*, *Phys. Rev.* **D 58** (1998) 021302 [hep-ph/9709417];
S. Tsujikawa, K. Maeda and T. Torii, *Resonant particle production with non-minimally coupled scalar fields in preheating after inflation*, *Phys. Rev.* **D 60** (1999) 063515 [hep-ph/9901306];
S. Tsujikawa, K. Maeda and T. Torii, *Preheating with non-minimally coupled scalar fields in higher-curvature inflation models*, *Phys. Rev.* **D 60** (1999) 123505 [hep-ph/9906501].

[57] V. Sahni and S. Habib, *Does inflationary particle production suggest $\Omega_m < 1$?*, *Phys. Rev. Lett.* **81** (1998) 1766 [hep-ph/9808204].

JHEP08(2000)019